SPONSORED RESEARCH GRANT APPLICATION

Dr. Brian L. Walter

October 17, 2005

I am applying for a Sponsored Research Grant to continue the research program that I began with my doctoral dissertation and that I have continued to pursue in my three years at The Evergreen State College. My goal for next summer is to complete my solution to the problem I approached in my dissertation, a classification problem for directed graph algebras. I will describe my work and its relevance in some detail below.

MY RESEARCH

To begin, let us say a few words about directed graphs. Directed graphs are natural mathematical structures that arise in various forms throughout the sciences; every flowchart, for example, is a directed graph. We can picture an arbitrary directed graph as a collection of dots together with arrows leading from some dots to other dots. Directed graphs have long been studied in detail in Mathematics, and they have particular relevance in the field of Computer Science, since programs can be represented partly in terms of directed graphs. My research will expand current knowledge about directed graphs and answer some longstanding questions about them.

Given any mathematical structure, in particular a directed graph, it is frequently useful to define an *operation* using the natural properties of that structure. In the case of directed graphs, we use the arrows to define a multiplication operation, and what results is a *directed graph algebra*. (Here algebra is a technical term; we simply use it to mean a collection of objects together with one or more operations that act on those objects.) The properties of a structure are generally mirrored quite nicely in the algebra that results from it, so investigating the algebra's properties frequently leads to insights about the structure itself. My work focuses on the algebraic properties of directed graphs.

In any given algebra (and, in particular, in every directed graph algebra), certain laws hold; for example, if we consider the collection of all real numbers to form an algebra, together with the operation +, then one law that we can express and easily verify is the commutative law for addition, $x+y = y+x$. Laws provide a nice window into the inner workings of an algebra, and sometimes writing down a few laws tells us everything that there is to know about a particular algebra.

When finitely many laws suffice to express everything true about an algebra, we say the algebra is *finitely based*. It is natural to believe that every finite algebra is finitely based; how could it be otherwise, since the algebra itself is finite? It is therefore a surprise that there are many known examples, some of them first published in my dissertation, of *non*-finitely based finite algebras; these "pathological" algebras require us to write down an infinite list of laws.

The finite basis classification problem, then, is the problem of distinguishing the finitely based finite algebras from the non-finitely based ones. My dissertation gave half of such a classification in the case of directed graph algebras. In it, I developed several theoretical tools to use in classifying algebras, and I successfully used those tools to classify a major subclass of directed graph algebras, namely, those in which there is an arrow from every dot to itself. My work since coming to Evergreen has been fruitful: I have continued to develop and apply my methods, discovering several new infinite classes of finitely based directed graph algebras; I have written a paper (which I soon expect to hear has been accepted for publication) and have a second paper in progress; and I was invited to give a talk at a special session at the American Mathematical Society Southeastern Section Meeting at Vanderbilt University in October 2004. The work is going well, when I have time to pursue it. My goal for next summer is to finally completely solve this classification problem, to classify all of the directed graph algebras.

Regarding interdisciplinarity: my research bridges the gap between three large areas of Mathematics - Logic, Algebra, and Graph Theory - and also serves as a point of contact with other disciplines that use graphs, like Computer Science. That being the case, I am able to interact fruitfully about my research with Mathematicians in many subfields of Mathematics as well as researchers in other areas, such as Computer Science, Biology, and Physics.

THE RELATIONSHIP OF MY RESEARCH TO THE COLLEGE

While I consider myself to be a teaching Mathematician (indeed, my initial desire to obtain a Ph.D. derived mostly from a desire to teach Mathematics at the college level), I also consider the continuation of my Mathematics research to be vital to my work as a member of the academic community. I chose to work at Evergreen partly because I feel that the balance of teaching vs. research here is a good fit with my priorities. In the past three years, I have kept at my Mathematics research while focusing primarily on teaching, and I plan to work to maintain this particular balance between teaching and research.

That said, I feel that it is particularly important for me to finish my classification of the directed graph algebras now, at the beginning of my academic career, while the work is still fresh and while interest in my results is high. After my talk at Vanderbilt last October I received very positive feedback from a number of important researchers in my field. They and others working in this area are eager to see the results of this research program because general methods for classifying algebras are few and the classification problem is not well understood; I am one of the few to make progress on a problem of this sort.

Next summer's research will lead to both conference talks and the publication of an academic paper in an established journal of Mathematics. The paper and talks will be good exposure for The Evergreen State College in the world of research Mathematics, raising awareness of the school as a place where scholarship is conducted. My work will also serve as a topic for continued dialogue with my colleagues and even with highly motivated students. This year I am fortunate enough to have a talented undergraduate working with me in my research area; with time to devote exclusively to research, I will be able to develop more subproblems of a suitable size for future undergraduate research students to tackle.

DETAILED PLANS

My request is for funding for six weeks of paid research during Summer 2006. My sense is that I am close to completing the work described above. Naturally, there's no telling if I'm right about that until I do the work; I might find the answer after a week, or it might still be months of work away. Still, some progress that I made last summer was very encouraging, and I believe that my recent results are pointing the way to a final solution. In the event that I do not need all six weeks to complete this proposed work, I will spend any remaining time writing up my results for publication.

The actual work of performing my research consists of reading journal articles, interacting (via e-mail) with colleagues in my research area, writing computer programs, and spending time thinking and experimenting on paper. The bulk of my research time is spent on this last; breakthroughs in my work have traditionally come after long hours of intense thought and careful study of examples. While I have occasionally been able to make some progress in writing up past research results during the school year, the particular demands of teaching at Evergreen do not seem to permit the kind of reflection necessary for me to make new advances in my current research. Sponsored research next summer would free me from the obligation of teaching to support myself, providing me the time and clarity to further establish and develop my research career. At present, I do not have any outside sources of funding for this research project.

I have arranged to have a letter of support for this application sent from Dr. Kirby Baker, my graduate advisor at UCLA, and I have attached a letter from Vauhn Foster-Grahler, the director of the Quantitative and Symbolic Reasoning Center at Evergreen. I am also including a current copy of my curriculum vitae, a brief report on the results of my previous sponsored research award (from Summer 2003), and a copy of the paper I wrote during that time. Thank you for your consideration.

Sincerely,

Brian L. Walter

Brian L. Walter

Attachments:

letter of support from Vauhn Foster-Grahler
Curriculum Vitae
Report on Summer 2003 Sponsored Research
copy of journal article ("The Finitely Based Varieties of Looped Directed Graph Algebras")

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DEPARTMENT OF MATHEMATICS
520 PORTOLA PLAZA
BOX 951555
LOS ANGELES, CA 90095-1555
PHONE: (310) 825-4701

October 28, 2005

Jeannie Chandler
Sponsored Research Grants
The Evergreen State College
2700 Evergreen Parkway NW
Olympia, WA 98505

Dear Ms. Chandler and Colleagues:

I am glad to have this opportunity to write on behalf of your faculty member Dr. Brian Walter, who is applying for sponsored research funding for the coming summer. Dr. Walter was my Ph.D. student, and I have also heard him speak on this work at a conference, so I am very familiar with the content of his proposal and his progress since his such grant four years ago.

As I have mentioned before, Dr. Walter is a self-starter in research, having demonstrated the capability of taking a problem and coming up with many interesting approaches to it on his own. He has continued to do that, even while investing much time and energy in his teaching.

A basic problem in modern algebra is to study the algebraic laws satisfied by different algebraic structures, and especially to be able to determine when finitely many laws suffice (the "finite basis problem"). In his research, Dr. Walter has been studying this question for so-called directed graph algebras– algebraic structures obtained by starting with a directed graph, supplementing it by an additional element, and imposing a binary operation determined by the graph. Such algebras are of interest to a number of researchers. Dr. Walter obtained significant results in his thesis. Then in his last sponsored summer research and later, Dr. Walter extended his work to cover large families of cases that had not previously been resolved. After he had spoken on these results at a conference at Vanderbilt, Prof. G. McNulty, a leader in algebra of this kind, remarked that he had spent a month on the same question and had failed to get anywhere towards the answers that Dr. Walter had found. Dr. Walter has submitted much of this work for publication.

I have full confidence in Dr. Walter's prospects for achieving further break-

throughs during sponsored summer research and in writing them up for publication. I recommend that he be supported.

Sincerely,

Kirby A. Baker

Kirby A. Baker
Professor of Mathematics Emeritus
UCLA

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360-867-5630

October 15, 2005

Dear committee members,

I strongly support Brian Walter's sponsored research proposal for four reasons. First, directed graphs are one of the most ubiquitous mathematics in the everyday world, yet there is a dearth of information about them. The work Brian proposes to do is very sophisticated mathematically, yet the results will have meaning for areas as diverse as economics, evolution, and anthropology. The idea that a faculty member from The Evergreen State College will contribute this work to mathematics is truly an opportunity that cannot be missed. Second, working in an area of mathematics that has such broad interdisciplinary potential increases the opportunity to include mathematics throughout the curriculum at Evergreen. Third, Brian's work will spawn many avenues of undergraduate research opportunities that will greatly enhance the college's ability to serve upper-division math students. Finally, in my work as Director of the Quantitative and Symbolic Reasoning Center I have had an opportunity to talk with many faculty members who teach mathematics at Evergreen. Without exception, those who I have talked with are eager to engage in discussions of the scholarly work each of us is doing in our discipline. Providing Brian the opportunity to do this research will be a wonderful gift to these faculty, the college, and to mathematics.

Brian's research is unique in that it is mathematics at its highest level yet it will have applications throughout the disciplines. The college's demanding teaching schedule makes it difficult to do research of this kind during the academic year. I believe that honoring Brian's request will be the only way he will have time to complete his work. Brian is an excellent mathematician and Evergreen is lucky to have him as a member of our faculty. I hope the college realizes Brian's value and awards him the opportunity to do this important and meaningful work.

Sincerely,

Vauhn E. Foster-Grahler
Member of the Faculty and Director of the QuaSR Center
The Evergreen State College

REPORT ON SUMMER 2003 SPONSORED RESEARCH

DR. BRIAN L. WALTER
OCTOBER 20, 2003

My six weeks of sponsored research in Summer 2003 gave me the opportunity to do a great deal of academic research in my area of Mathematics, Universal Algebra. During those six weeks, I finished preparing one journal article for publication, refined a great deal of earlier work on looped directed graph algebras, and obtained numerous new results about loop-free directed graph algebras.

I had finished a rough version of my first academic paper, *The Finitely Based Varieties of Looped Directed Graph Algebras*, before starting the six weeks of sponsored research, but getting back into my study of the subject required me to go over my earlier work. As I studied that work carefully, I saw several places in which the paper could be improved; I tightened up most of the proofs, extended some results, and obtained a significant new result to include, making the paper relevant to a larger mathematical audience. Working with my colleague Dr. Kirby Baker at UCLA, I turned the rough version into a finished product, ready for publication. I will submit it to Acta Scientiarum Mathematicarum, a refereed Mathematics journal, as soon as I get a last round of comments from Dr. Baker. The current version of the paper is available online at `http://grace.evergreen.edu/~bwalter/paper.pdf`.

Having largely finished the paper, I turned my attention to the next topic at hand, a classification of the finitely based varieties of loop-free directed graph algebras. The loop-free directed graph algebras turned out to present more difficulties than the looped ones; while there are only 5 finitely based varieties of looped directed graph algebras, I showed that there are infinitely many loop-free such varieties, making the loop-free case a much knottier problem. Nonetheless, I made progress toward a characterization, completely characterizing the cyclic loop-free directed graph varieties and making inroads into the acyclic case.

I did not achieve one of the stated goals of my project, which was to finish the classification problem for directed graph algebras in general; the problem was harder than anticipated, as research problems in Mathematics are wont to be. However, I achieved excellent results in the refinement of my earlier work and made great strides forward on the more general research problem. More importantly, I got my research program going again, which, for me, was the point of the project; I am now in a position to work steadily to resolve the remaining cases and obtain a complete characterization of the directed graph algebras (my demanding teaching schedule permitting).